HEAT TRANSFER WITH ICE – WATER MIXTURE FLOWING IN A PIPE

S. I. Isataev and N. V. Masleeva

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The forced motion of an ice and water mixture in the main section of a round pipe is investigated. The temperature profiles and heat transfer are determined as a function of mixture concentration.

Heat transfer has been investigated in [1-3] for particles suspended in a flow with motion of a two-phase (sludge) medium within a closed volume and in pipelines. Heat transfer was considered for solid suspended particles of constant mass. In [4-8], heat transfer was investigated for the boundary layer in the presence of injection or withdrawal of liquid (and with melting) at the surface past which the flow moves.

We shall assume that we have a steady two-phase flow within a round pipe; the solid phase takes the form of spherical particles of identical diameter. The specific heat flux through the pipe wall is constant. The discussion commences with the assumption that the thermal boundary layers join up. We also assume that the liquid phase moves at a certain velocity that is constant over a cross section (u = const), while the rate of motion of the solid phase may differ by a certain constant value. We consider the case of moderate heat fluxes, where the change in concentration along the pipe can be neglected in first approximation. We also assume that the temperature of a solid particle is constant throughout the entire volume, and equal to the melting point.

The energy equation for this case has the form

$$u\frac{\partial\Delta T}{\partial x}=\frac{\lambda}{\rho c_p}\left(\frac{\partial^2\Delta T}{\partial r^2}+\frac{1}{r}\frac{\partial\Delta T}{\partial r}\right)-\frac{q_1\Delta T}{\rho c_p(1-C)}. \tag{1}$$

We use the following conditions to solve (1):

$$\Delta T=\Delta T_{w0}\ \text{for}\ \ x=0\ \text{and}\ r=R, \tag{2}$$

$$\frac{\partial\Delta T}{\partial r}=0\quad\text{for}\quad r=0. \tag{3}$$

Setting up the heat-balance equation for the pipe section of length Δx, we obtain one more relationship:

$$q_wR=c_p\rho(1-C)u\frac{d}{dx}\int_0^R\Delta Trdr+q_1\int_0^R\Delta Trdr. \tag{4}$$

We seek a solution in the form

$$\Delta T=(T-T_w)+(T_w-T_{\mathrm{m}})=f_1(r)+f_2(x). \tag{5}$$

Substituting (5) into the initial equation and using (2)-(4), we obtain the solution

$$\Delta T=\frac{q_wR}{\lambda(1-C)}\,\frac{I_0(z)-\exp(-B)}{\int_0^z I_0(z)\,zdz}, \tag{6}$$

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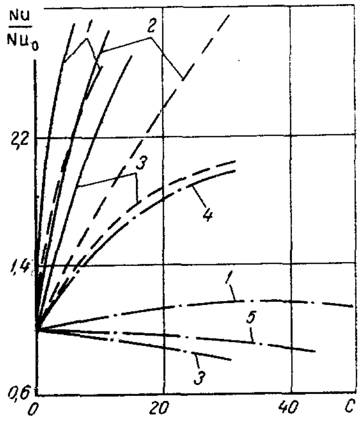

Fig. 1

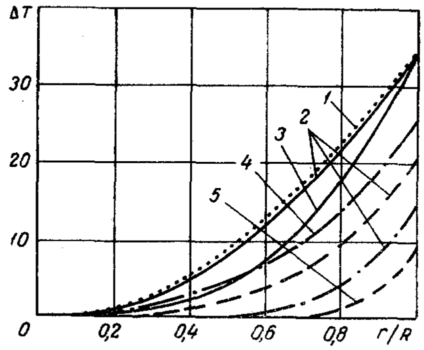

Fig. 2

Fig. 1. Relative Nusselt number for a pipe as function of concentration (C, %) [solid lines) Nu_1 = 70; dashed lines) 35; dash-dot lines) 2]: 1) d = 0.010; 2) 0.016; 3) 0.020; 4) 0.005; 5) 0.015 m.

Fig. 2. Temperature profiles (°K) for zero section (x = 0) [solid lines) Nu_1 = 2; dash-dot lines) 35; dashed lines) 70]: 1) water; concentration of solid particles: 2) 5%; 3) 40; 4) 2; 5) 20.

where

$$z = \sqrt{\frac{q_1}{(1-C)\lambda}}\, r \quad (z = Z \text{ for } r = R), \tag{7}$$

and

$$B = \frac{\lambda Z^2 x}{\rho c_p u R^2}\,; \tag{8}$$

here $I_0(z)$ is a modified zero-order Bessel function of the first kind.

We determine the specific sink power,

$$q = q_1 \Delta T = n\alpha_1 F_1 \Delta T \quad (F_1 = \pi d^2). \tag{9}$$

To determine the particle-surface–medium heat-transfer coefficient, we write the following relationship (the number of particles in unit volume in terms of the concentration):

$$q_1 = 6C\,\frac{\mathrm{Nu}_1 \lambda}{d^2}\,. \tag{10}$$

We now find the dimensionless pipe-wall–medium heat-transfer coefficient:

$$\mathrm{Nu} = \frac{q_w 2R}{(T - T_w)\lambda}\,. \tag{11}$$

Using (6) we finally obtain

$$\mathrm{Nu} = (1 - C)\,\frac{2\int_0^Z I_0(z)\,z\,dz}{I_0(Z) - \frac{2}{Z^2}\int_0^Z I_0(z)\,z\,dz}\,. \tag{12}$$

In the limit, where the concentration of solid phase equals zero, Eq. (6) becomes

$$\Delta T = \frac{2q_w x}{\rho c_p R u} + \frac{q_w R}{2\lambda}\,\frac{r^2}{R^2}\,, \tag{13}$$

and the heat transfer to pure water is determined by the Nusselt number $\mathrm{Nu}_0 = 8$.

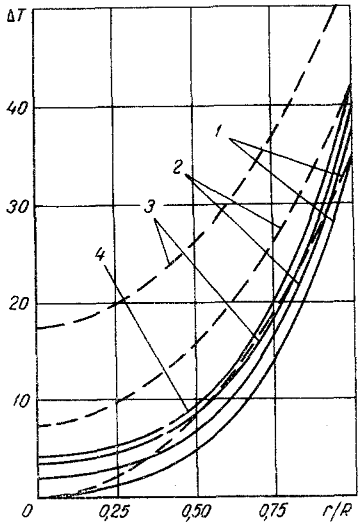

Fig. 3. Temperature profiles (ΔT, °K) for various pipe sections [dashed lines) water; solid lines) C = 40%]: 1) x = 0; 2) 83 m; 3) 200 m; 4) ∞;

To illustrate the influence of the solid-phase concentration on the temperature distribution over the pipe cross sections and on the channel-wall–mixture heat-transfer coefficient, we carried out the calculations for a water–ice mixture.

The computations were carried out for the following values: R = 0.021 m, d = 0.016 m, u = 0.500 m/sec, q_w = 1886 J/m^2 · sec.

In actual flow of sludge, the solid particles can move at a certain rate relative to the liquid (owing to Archimedian buoyancy, etc.); thus we selected a Nusselt number of 2 for the particles (the relative particle velocity is zero), 35 and 70. Allowance was made for the effect of particle melting on the heat-transfer coefficient by the method employed in [7].

Figure 1 shows the pipe relative Nusselt number for various particle heat-transfer coefficients and diameters. As we see, an increase in the solid-phase particle Nusselt number increases the heat-transfer coefficient for the pipe, as is quite natural. Moreover, the heat-transfer coefficient depends on the particle diameter. For the same concentration, fine particles have a greater melting surface than large particles; thus there is more vigorous exchange of heat between the sludge and the channel walls.

Figure 2 shows temperature profiles for various values of Nu_1 and solid-phase concentrations for 16 mm diameters at the zero (c = 0) section of the channel. As we might expect, with increasing concentration and Nu_1, the temperature gradient increases at the pipe wall, while the mean temperature of the liquid drops over the pipe cross section.

In contrast to a flow of water, the presence of constant-concentration sludge produces a situation such that as the x coordinate increases, the mixture temperature does not rise to infinity, but approaches a certain limiting value (Fig. 3). The resulting solution is approximate, since in actuality the sludge concentration varies both along the pipe length and cross section, while the flow velocity profiles differ from those used in the problem.

The computational results do give a correct qualitative picture of influence of sludge concentration on the temperature profile and of the intensity of heat exchange between the sludge flow and the channel walls.

NOTATION

R	is the pipe radius, m;
d	is the particle diameter, m;
u	is the liquid-phase flow rate, m/sec;
q_w	is the specific heat flux from the wall, W/m^2 · deg;
$C = n\pi d/6$	is the volume concentration of solid phase;
n	is the number of particles in unit volume, 1/m^3;
T	is the liquid temperature, °K;
T_m	is the melting point, °K;
$\Delta T = T - T_m$;	
T_w	is the temperature of the liquid at the pipe wall, °K;
x	is the coordinate along the tube axis, measured from the point at which the thermal boundary layers merge;
λ	is the thermal conductivity coefficient, W/m · deg;
ρ	is the density of the liquid-phase material, kg/m^3;
c_p	is the specific heat capacity of the liquid, J/kg · deg;
r	is the coordinate measured from the axis along the pipe radius, m;
q	is the specific sink power, W/m^3;
$q_1 = q/\Delta T$, W/m^3 · deg;	
α_1	is the particle-surface–liquid heat-transfer coefficient, W/m^3· deg;
F	is the particle surface area, m^2;

$Nu_1 = \alpha_1 d/\lambda$	is the Nusselt number for the particle;
$Nu = \alpha 2R/\lambda$	is the Nusselt number for the pipe;
$\alpha = q_W/\overline{(T - T_W)}$	is the pipe-wall–sludge heat-transfer coefficient, $W/m^2 \cdot deg$;
$\overline{(T - T_W)}$	is the difference in the temperatures in the liquid and wall, averaged over the pipe cross section, °K;
Nu_0	is the Nusselt number for the pipe with C = 0.

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